Stochastic Modelling to Identify Spatial and Temporal Changes in Rainfall Pattern Along the South West Coast of Kanyakumari District, India

S. Kaliraj, N. Chandrasekar and N.S. Magesh

Abstract--- *In meteorology concept, rainfall is a dynamic phenomenon and it is vary from place to place. Rainfall is a major source for recharging the fresh water in both surface and sub surface of the earth. This study is to explore the changes in rainfall pattern in past, present from the historical rainfall series used to predict the probability of rainfall amount for subsequent years in future using the stochastic model along the coastal area of Kanyakumari district, Tamilnadu. Rainfall data series were collected from 28 rain gauge station located across the study area for the period of 35 years between 1971 and 2006. Stochastic modelling technique combined with transition matrix and frequency analysis has been introduced to execute the average annual rainfall data to drawn the spatial and temporal changes of rainfall amount at all stations according to the stochastic process and that provided the relative and cumulative probability class interval of rainfall for years in future. Each class is treated as states of frequency and the corresponding probability values are considered and assigned to sets of defined year. From the prediction of rainfall, it is estimated that the probable amount of rainfall is 401 mm – 509 mm for the period of 2012 and 2014 and the amount of rainfall in 2013 is 901 mm – 1009 mm.*

Keywords--- *Average Annual Rainfall, Spatial and Temporal Changes, Markov-Chain Model, Rainfall Prediction, Transition Matrix, Southwest Coast, India*

I. Introduction

RAINFALL has a vital role in equal balancing of surface and sub-surface water resources through the hydrologic cycle process in nature [1].

The amount of rainfall level is varying in time to time which are influenced by climatic condition of the local area. Hence, prediction of rainfall is a complex problem to meteorologist till date. Understanding the spatial and temporal variation in rainfall level and intensity is important to forecast the trends of rainfall in future [2]. Spatial and temporal changes of rainfall pattern have been estimated from the historical rainfall data using statistical methods and can provided reliable output [3]. Forecasting the rainfall process has performed by number of Present days, a numbers of statistical methods such as Mann-Kendall test [4], Stochastic model [5], Markov-Chain method [6] these methods are facilitated effective mathematical computation to find spatial and temporal changes in rainfall pattern, intensity and concentration of an area.

The probability of rainfall of an area at the defined period was estimated from the long time series of rainfall data using transition probability matrix based stochastic model and it is also has been used to predict the rainfall pattern in future using statistical analysis of standard deviations and autocorrelation coefficient [7]. According to reference [8] the first order the Markov-Chain model and transition probability matrix were used to compute the relative and cumulative frequency and probability distribution of rainfall amount of an area [9], [10].

In this study, the stochastic model Marchov-Chain method were applied successfully to drawn the following process in prediction of rainfall probability such as the spatial and temporal changes in average annual rainfall pattern and probability estimation of rainfall during the present period and also to predict the future trend and to identify the dynamic change of rainfall. Further, GIS processes also used to interpret the spatial and temporal variation in rainfall pattern along the study area.

II. Location of Study Area

The present study probability estimation has been carried out in South-West coast of Kanyakumari district in Tamilnadu, India. The geographical extent of this area is 77^{o} 16’ 36.29’’E to 77^{o} 34’ 27.08’’E longitude and 8^{o} 03’ 22.45’’N to 8^{o} 10’ 14.45’’N latitude. The area has comprised with different types of morphological landforms of such as beach, sand dunes, vegetative cover, shallow lands, settlements and other manmade infrastructure. The area has occupied 17.2 % of total population of the district. The whole area is experiencing the moderate subtropical climate from last two decades with an average annual rainfall of 14.65 cm and temperature ranges from 24^{o}C to 26^{o}C. From the 35 years of rainfall series between 1971 and 2006, it is observed that highest rainfall amount is 26.14 cm in 1979 and least amount is 8.83 cm in 2003.

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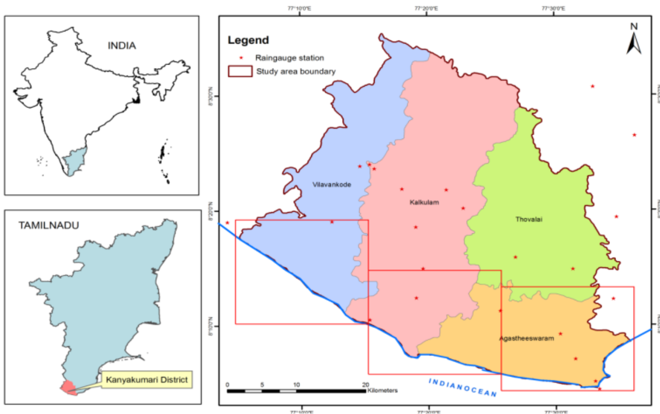

Figure 1: Location of the Study Area

III. Materials and Methods

The series of rainfall data were collected from 28 rain gauge stations of Indian Meteorological Department located across the study area for 35 years between 1971 and 2006. The base map layer was extracted from Survey of India topographical map (scale 1: 25000) and the locations of all stations were pointed out based on their corresponding geographical coordinates.

In the study, stochastic model is used to analysis the long time series of rainfall data. This is a time series analysis of randomly distributed sample data for forecasting the future trends of variables of the sampled area. In the study, number of statistical analysis such as standard deviation, transition matrix, and Markov-Chain model were introduced to derive the spatial and temporal changes in rainfall and also predict the probability of rainfall level in future.

A. *Annual Average Rainfall Calculation*

The annual average rainfall is derived from the rainfall data which were recorded in 28 rain gauge stations using the following equation represented as

$$AR = \frac{MR1+MR2+\cdots MRn}{RN} \qquad (1)$$

Where, AR = average annual Rainfall, MR = sum of daily rainfall of a month, RN = total number of months.

B. *Frequency Distribution of Average Rainfall*

From the above equation, average annual rainfall was calculated for each year and the value is substituted into equation 2 to determine number of class interval and the range of class interval is prepared from maximum and minimum values of rainfall divided by number of class interval to identify the suitable frequency distribution table using the equation 3 as follows,

$$\text{Class interval } (k) = 1 + 3.3 * \log_{10} N \qquad (2)$$

Where; k = number of class interval, N = total number of Rain gauge station

$$f = \frac{Range\ in\ Rainfall\ (Max - Min)}{k} \qquad (3)$$

From the mathematical processes using above equations, it is calculated the class interval (k) value is 5 and the range of class interval is 10. So that the frequency of rainfall is computed according to the range of class interval and each class assigned by specific states of rainfall such as A, B, C, D, E, F and G. The frequency distribution of rainfall for all states is shown in Table 1.

Table 1: Frequency Distribution of Average Annual Rainfall between 1971 and 2006

Range of Class Interval of rainfall (mm)	States	Frequency
40.10 – 50.09	A	1
50.10 – 60.09	B	2
60.10 – 70.09	C	5
70.10 – 80.09	D	10
80.10 – 90.09	E	8
90.10 – 100.09	F	4
100.10 – 110.09	G	6

It is described that the highest rainfall level is 100.10 mm – 110.09 mm is having frequency values 6 and is denoted by state G, the least amount of rainfall is 40 mm to 50 mm recorded in single frequency and state D with range of rainfall is 70.10 mm – 80.09 mm that have frequency is 10.

C. *Markov-Chain Model of Rainfall Data Analysis*

The First order Markov-Chain model is proposed by Markov [10]. The first order Markov-Chain model is used to estimate transition probability between the states of successive time instances. In this method, the analyses of random series of variables were carried out by stochastic process and random process and the ordered set of related random variables *X (t)* where, *X* is variable and *t* is a time or period of data observed. The frequency of state against rows and columns was applied into transitional probability matrices of various time steps for estimating the probability of rainfall pattern. A first order Markov chain is a stochastic process; it is having the property of state frequency distribution of rainfall at time *t*, *X t* (defined year of future). Hence, the transition probability matrix constructed by applied states in row and column (*n* x *n)* of matrix and transitions are made between two successive time instances. It is then possible to find the number of transition probabilities of each row against column (P_{ij}) from a state at time *t* to another state at time *t+1*. By following this, transition probability matrix (P_t), (*t+1*) can be prepared from frequency distribution table (table 1). The structure of the transition probability matrix is expressed as,

$$P_{t,\,t+1} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & . & . & . & P_{1n} \\ P_{21} & P_{22} & P_{23} & . & . & . & P_{2n} \\ P_{31} & P_{32} & P_{33} & . & . & . & P_{3n} \\ P_{41} & P_{42} & P_{43} & . & . & . & P_{4n} \\ P_{51} & P_{52} & P_{53} & . & . & . & P_{5n} \\ P_{61} & P_{62} & P_{63} & . & . & . & P_{6n} \\ P_{71} & P_{72} & P_{73} & . & . & . & P_{7n} \end{bmatrix}$$

In the above transition probability matrix, the probability of rainfall of states *i* at time (*t*) and the state *j* at time (*t+1*) can be estimated using the equation given below [11]

$$P_{ij} = \frac{f_{ij}}{\sum_{j=1}^{c} f_{ij}} \qquad i, j = 1, 2, 3 \ldots, C \tag{5}$$

Where, $P_{i\,j}$ is state of average annual rainfall at row (*i*) and column (*j*), $f_{i\,j}$ is historical frequency of transition probability from the state row *i* and state *j*, and *C* is the maximum number of state. From the transition probability matrix, the yearly average annual rainfall value drawn into matrix for estimating transition probability of rainfall. The state of rainfall at *i* and *j* up to state *n*, the probabilities are validated by the statistical definition in that any state with probability exactly between zero to one, it is represented as, $0 < P_{i\,j} < 1$, where, i, j is state *1, 2, 3, ...C*. In the execution of transition matrix, the matrix elements $P_{i\,j}$ are constituted for calculating relative probability of the measured rainfall state that placed at *i* at the time *t+1* and state *j* at previous time step *t*. Successive multiplications of state in all elements ($P_{t,\ t+1}$) of matrix performed by itself until a complete transition probabilities for given time series using first order Markov-Chain model.

The relative transition probability values in row *i* and column *j* at the state *k* are applied to calculate the cumulative probability (Pik) using the following formula expressed as,

$$P_{ik} = \sum_{j=1}^{k} P_{ij} \tag{6}$$

The relative probability parameters within the cumulative probability transition matrix are complied with the definition, in that the value of each row ends with 1. Hence, cumulative summation of values within each row leads to generate the synthetic series of rainfall in states between the ranges of uniform random number (0 to 1) with the help of random number generator.

IV. Results and Discussion

A. *Fluctuation of Average Annual Rainfall*

Average annual rainfall for 35 years was derived from the long time rainfall data of all rain gauge stations and the rate of annual changes was interpolated in Figure 2. This is depicted that the spatial distribution of average annual rainfall is high (2684 mm) in the west and middle part and the east part is experienced by least amount of rainfall (883 mm). Among the period of 1971 and 2006, the locations namely Pechipparai and Surakkod are received high amount of rainfall and the rainfall level is least amount in Kottaram, Aralvoimozhi and Kanyakumari.

The spatial distribution of rainfall along the study area is in different amount and in different period. The rainfall level is decreased gradually between the periods of 1971 and 1986, and in the next subsequent periods were recorded significant amount of rainfall till 2003. It is increased positive wiz, with the amount of average rainfall of 83.65 mm 99.61 mm and 94.90 mm during the years of 2004, 2005 and 2006 respectively.

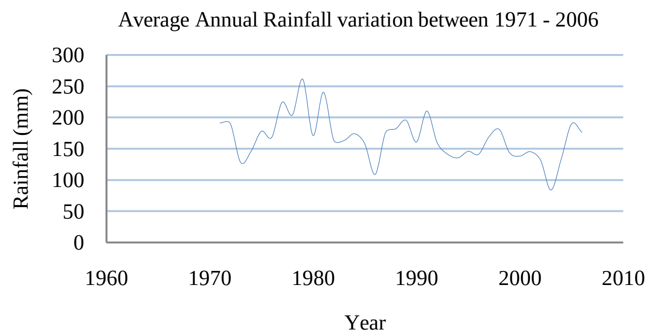

Figure 2: Fluctuation of Average Annual Rainfall

B. *Probability of Rainfall prediction*

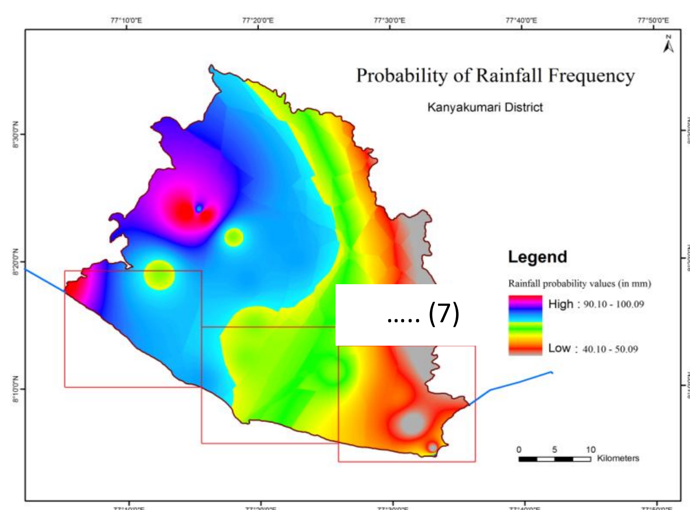

Figure 3: Probability Distribution of Rainfall for Future

The result of transition matrix was produced cumulative probability rainfall for each state and the uniform random numbers were derived between the range of 0 and 1 using random number generator. The cumulative probability values are segmented to different classes according to the derived random uniform numbers within the random number rang and the states are assigned to corresponding random uniform number, then rainfall probability values of each year was assumed range of class interval of annual rainfall. Hence, the synthetic series of random rainfall pattern composed by both uniform random number and state (Table 2).

The series of uniform random numbers are represented by subsequent years in future (later few years i.e. 2012, 2013, and

2014) and the rainfall pattern of these years are predicted from the corresponding state's frequency of average annual rainfall which is given in table 1. The uniform random numbers, states with its rainfall frequency are represented in table 2. From the table, the random numbers denoted state B for 2012 and 2014 and it is predicted that the range of rainfall frequency is 50.10 to 60.09 mm for the same and the state F is assigned to 2013, it is equal to the rainfall is 90.10 to 100.09 mm, likewise the all the rain gauge points added to attributes of predicted rainfall probabilities for ensuring the spatial distribution of predicted rainfall range in future using GIS environment (Figure 3).

Table 2: Probability of Rainfall Level Prediction for Future

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014
Uniform Random Number	0.41	0.997	0.720	0.933	0.001	0.128	0.302	0.999	0.146
State	C	F	D	F	A	B	B	F	B
Probability of Rainfall (mm)	60.1 - 70.0	90.1 - 100.09	70.10 - 80.09	90.10 - 100.09	40.10 - 50.09	50.10 - 60.09	50.10 - 60.09	90.10 - 100.09	50.10 - 60.09

V. Conclusion

The information on meteorological phenomenon such as rainfall status at present and for future periods may be predicted from the historical series of datasets using statistical method has a significant role in planning and management. Once, the rainfall prediction process is modeled adequately and appropriately then it is to be used in agricultural planning and flood prediction and so on. The analysis of series of annual rainfall data using stochastic process of first order markov-chain model is an alternative model of rainfall status forecasting. The model is proposed for rainfall pattern forecasting studies. In this context, the first order markov-chain method and transition probability matrix were used for calculating the relative and cumulative probability of rainfall, and it required only minimal computational steps to derive the result. Moreover, GIS environment is involved to add attributes of all rain gauge points for ensuring the spatial distribution of predicted rainfall range in future.

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